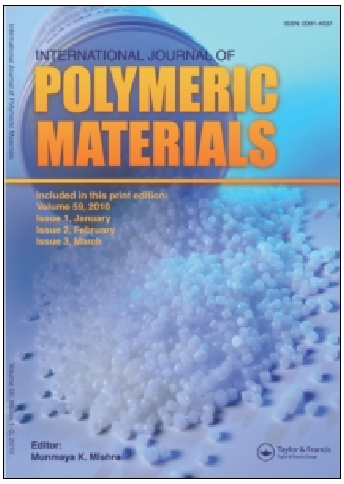

International Journal of Polymeric Materials
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713647664

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To cite this Article Wilczynski, Andrew P.(1977) 'Rigidity of Foamed Polymers', International Journal of Polymeric Materials, 5: 4, 291 — 294

To link to this Article: DOI: 10.1080/00914037708075212

URL: http://dx.doi.org/10.1080/00914037708075212

Intern. J. Polymeric Mater., 1977, Vol. 5, pp. 291–294

Printed in Reading, England

Rigidity of Foamed Polymers

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This paper presents utilization of Goodier's theory of inclusions for a special case of voids in matrix material. The problem has been treated numerically as analytical relations are of very complex nature. Final results, giving values of ratios of Young's moduli and Poisson's ratio and that of matrix material have been tabulated.

INTRODUCTION

Some commercially manufactured polymers used in bulk quantities as load carrying materials can, for the sake of weight and cost, be foamed to some extent.

It is important that the effect of foaming on the stiffness of the material should be understood and a solution to this problem is proposed in this paper. The solution assumes that there is no change in the surface energy of the foamed material and its validity has been demonstrated for low level of foaming.

THEORETICAL

A solution to the problem for elastic materials was presented by Goodier[1] in 1934, based on the theory of potential. This approach is equally applicable to linear viscoelasticity providing only that the material constants are replaced by the appropriate time dependant functions. For the case when time of loading is negligible compared with the time under load of the structure the values obtained from creep or stress relaxation experiments could be used. The approach of Goodier was concerned with inclusions and voids and this paper is an extension of that idea.

Consider a spherical cavity surrounded by a sphere of matrix material with material constants G and ν. The radii of void and matrix spheres r and R respectively must be selected in such a way that the ratio of the volume of the void to that of the external sphere is the same as the total void volume to that of the total volume of foamed polymer. This leads to the equation

$$f = \left(\frac{r}{R}\right)^3 = \frac{V_\nu}{V} \tag{1}$$

and represents the foaming coefficient, where V_ν is the volume of the voids. According to Goodier the stresses in the matrix may be written as follows

$$\begin{aligned}\sigma_{rr} = 2G\Big\{&\frac{1+\nu}{1-2\nu}H + F - \nu D\rho^2 + \frac{2A}{\rho^3} - \frac{2\nu}{1-2\nu}\frac{C}{\rho^3} + 12\frac{B}{\rho^5} + \\ &+ \left[3F - 3\nu D\rho^2 - \frac{2(5-\nu)}{1-2\nu}\frac{C}{\rho^3} - 24\frac{B}{\rho^5}\right]\cos 2\theta\Big\} \qquad (2)\\ \sigma_{rt} = -2G&\left[3F + (7+2\nu)D\rho^2 + \frac{2(1+\nu)}{1-2\nu}\frac{C}{\rho^3} - 24\frac{B}{\rho^5}\right]\sin 2\theta\end{aligned}$$

where A, B, C, D, H, F are arbitrary constants to be calculated and the directions r, t are as indicated in Figure 1.

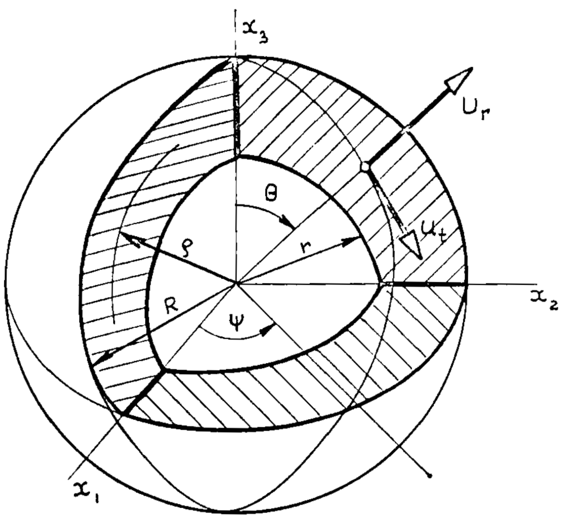

FIGURE 1

The displacement vector u_r can be found as

$$\begin{aligned}u_r = H\rho + F\rho + 2\nu D\rho^3 - \frac{A}{\rho^2} - \frac{3B}{\rho^4} + \\ + \left[3F\rho + 6\nu D\rho^3 + \frac{5-4\nu}{1-2\nu}\frac{C}{\rho^2} - g\frac{B}{\rho^4}\right]\cos 2\theta\end{aligned} \tag{3}$$

Assuming that the model is stressed in tension in the vertical direction the boundary conditions to satisfy are

$$\begin{aligned} \sigma_{rr}(R,\theta) &= \tfrac{1}{2}\sigma(1 + \cos 2\theta) \\ \sigma_{rt}(R,\theta) &= -\tfrac{1}{2}\sigma \sin 2\theta \\ \sigma_{rr}(r,\theta) &= \sigma_{rt}(r,\theta) = 0 \end{aligned} \tag{4}$$

where σ is the tensile stress applied to the material. The boundary conditions (4) lead to 6 linear algebraic equations from which the unknown constants A, B, C, D, H, F can be calculated. The analytical solution of this set of equations was disregarded and further simplification sought. Assuming that the constants are known it is possible to obtain a value for the displacement vector $u_r(R,\theta)$. By comparing this result with that obtained for a homogeneous material having material constants E_*,ν_*, the final results for the effective constants of the foamed material E_*,ν_*, can be expressed as functions of E, ν and f.

$$\begin{aligned} \frac{E_*}{E} &= \frac{\sigma \,.\, R}{u_r(R,0)} \\ \nu_* &= -\frac{u_r(R,\frac{1}{2}\pi)}{u_r(R,0)} \end{aligned} \tag{5}$$

These results, together with Eq. (4) were used for tabulation of ratios E_*/E and ν_* versus the foaming coefficient f. The value of ν was selected to be in the range 0.20(0.05)0.45 and the results are given in the following tables.

CONCLUSIONS

As can be seen from Tables I to VI the rigidity of a polymer can be changed over a wide range by foaming. It must be realized that these predictions may not be in good agreement with experimental results for some polymers. This is

TABLE I

$\nu = 0.20$

f	E_*/E	ν_*
0.1	0.6667	0.2450
0.2	0.4567	0.2762
0.3	0.3187	0.2961
0.4	0.2257	0.3071
0.5	0.1601	0.3107
0.6	0.1116	0.3070
0.7	0.0742	0.2935
0.8	0.0440	0.2612

TABLE II

$\nu = 0.25$

f	E_*/E	ν_*
0.1	0.6684	0.2805
0.2	0.4553	0.3020
0.3	0.3167	0.3151
0.4	0.2335	0.3209
0.5	0.1580	0.3202
0.6	0.1098	0.3123
0.7	0.0726	0.2940
0.8	0.0428	0.2546

TABLE III

$\nu = 0.30$

f	E_*/E	ν_*
0.1	0.6689	0.3165
0.2	0.4548	0.3285
0.3	0.3153	0.3349
0.4	0.2217	0.3357
0.5	0.1562	0.3308
0.5	0.1081	0.3190
0.7	0.0713	0.2962
0.8	0.0417	0.2505

TABLE IV

$\nu = 0.35$

f	E_*/E	ν_*
0.1	0.6705	0.3530
0.2	0.4551	0.3557
0.3	0.3146	0.3556
0.4	0.2204	0.3516
0.5	0.1547	0.3427
0.6	0.1067	0.3271
0.7	0.0700	0.3001
0.8	0.0485	0.2485

TABLE V

$\nu = 0.40$

f	E_*/E	ν_*
0.1	0.6731	0.3903
0.2	0.4565	0.3840
0.3	0.3146	0.3775
0.4	0.2196	0.3685
0.5	0.1535	0.3558
0.6	0.1055	0.3365
0.7	0.0690	0.3056
0.8	0.0403	0.2486

TABLE VI

$\nu = 0.40$

f	E_*/E	ν_*
0.1	0.6770	0.4286
0.2	0.4592	0.4137
0.3	0.3155	0.4009
0.4	0.2194	0.3572
0.5	0.1527	0.3705
0.6	0.1045	0.3476
0.7	0.0681	0.3128
0.8	0.0393	0.2507

because the internal structure of some molecules will change in the vicinity of a free surface. Such effects can be described and treated approximately by considering the surface energy of foamed and unfoamed polymers. Furthermore stress concentration factors and strength properties of foamed polymers could be obtained accordingly to the described method. This would depend upon strength criteria which are not sufficiently established at present.

Reference

1. J. N. Goodier, Concentration of stress around spherical and cylindrical inclusions and flaws, *J. Appl. Mech.* **55**, 39 (1934).